جَـامعة
المَـنارة
MANARA UNIVERSITY

Steel Structures 2
Sem. 1
2024-2025

أ.د. نايل محمد حسن

Lecture 9-10

- Flexural Members

- ✓ -I- Laterally Restrained Beams
- ✓ II- Laterally Unrestrained Beams

- Beam-Column Members

Beam-Column Members

Introduction: Beam-Column Members

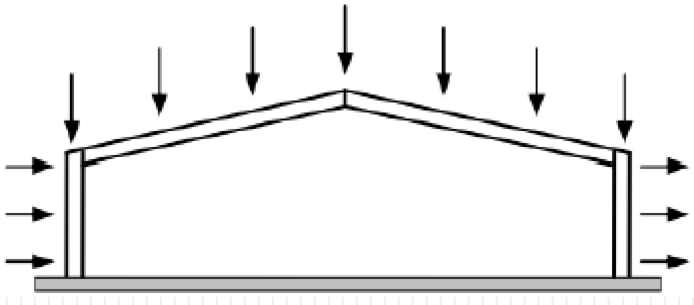
- **Axial force members are, in practice, subjected to axial load as well as bending in either or both the axis of the cross section.**

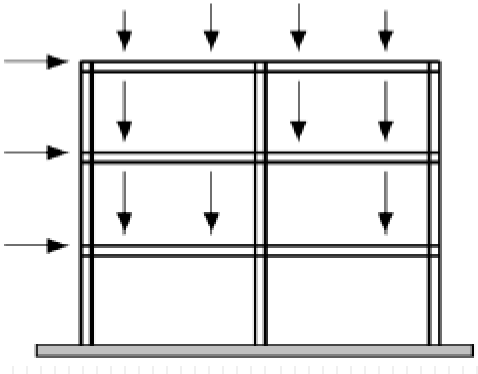
- **Similarly flexural members may also be subjected to axial load.**

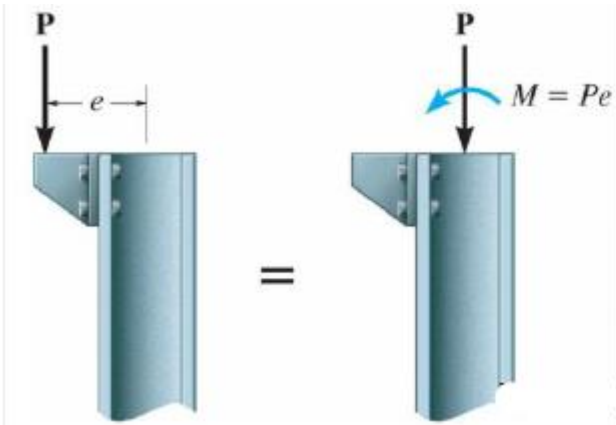
- **In either case, a member subjected to both significant axial and bending stresses is termed as Beam-Column Members.**

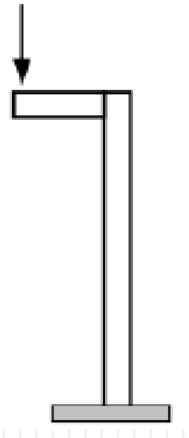
- **The behavior of such members results from the combination of both effects and varies with slenderness.**

Introduction: Beam-Column Members









A member subjected to both significant axial and bending stresses is termed as Beam-Column Members.

Introduction: Beam-Column Members

- At low slenderness, the cross sectional resistance dominates.
- With increasing slenderness, pronounced second-order effects appear, significantly influenced by both geometrical imperfections and residual stresses.
- At high slenderness range, buckling is dominated by elastic behavior, failure tending to occur by flexural buckling (typical of members in pure compression) or by lateral-torsional buckling (typical of members in bending).
  - The behavior of a member under bending and axial force results from the interaction between instability and plasticity and is influenced by geometrical and material imperfections. Therefore very complex.

Introduction: Beam-Column Members

The verification of the safety of members subject to bending and axial force is made in two steps:

- Verification of the resistance of cross sections .
- Verification of the member buckling resistance (in general governed by flexural or lateral-torsional buckling).

Cross section resistance

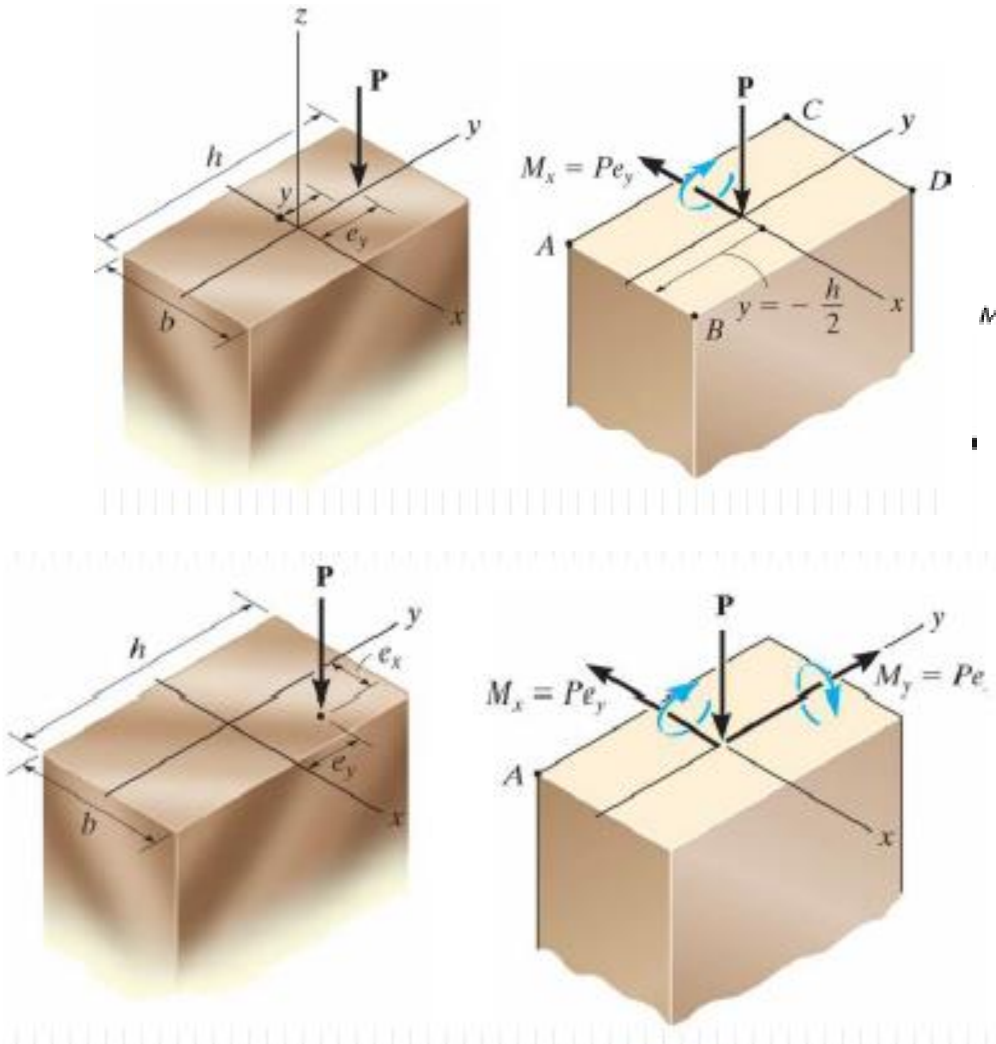
The cross section resistance is based;

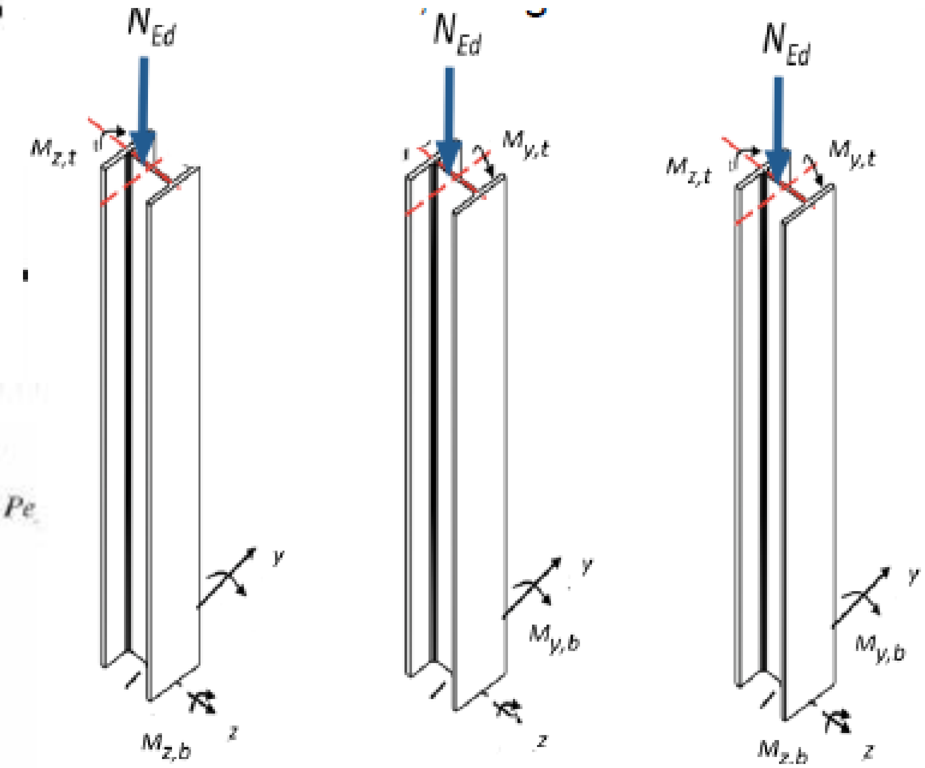
- **on its plastic capacity (class 1 or 2 sections) or**
- **on its elastic capacity (class 3 or 4 cross sections).**

When a cross section is subjected to bending moment and axial force ($N + M_y$, $N + M_z$ or even $N + M_y + M_z$),

the bending moment resistance should be reduced, using interaction formulas.

Cross Section Resistance : M-N interaction

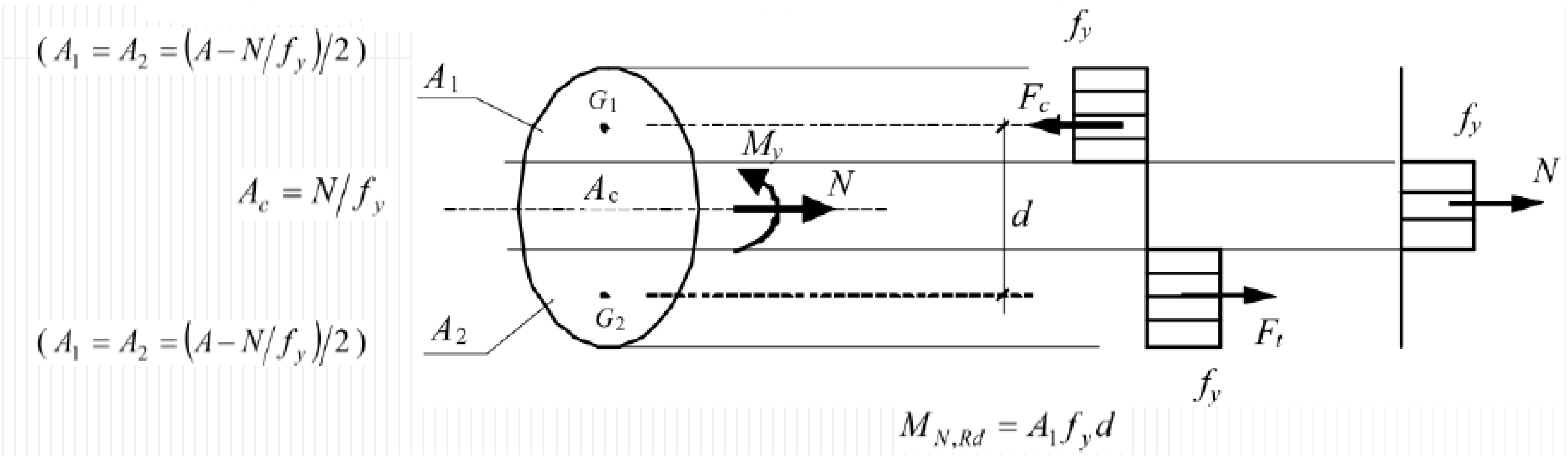




Cross Section Resistance : M-N interaction

- The interaction formulae to evaluate the elastic cross section capacity are the well known formulae of simple beam theory, valid for any type of cross section.
- However, the formulae to evaluate the plastic cross section capacity are specific for each cross section shape.
- For a cross section subjected to N + M, a general procedure may be established to evaluate the plastic bending moment resistance $M_{N.Rd}$, reduced by the presence of an axial force N.

Cross Section Resistance : M-N interaction



- **Although the interaction formulae are easy to obtain by applying the general method, the resulting formulae differ for each cross sectional shape and are often not straightforward to manipulate.**

Cross Section Resistance : M-N interaction

- **Historically, several approximate formulae have been developed, and, Villette (2004) proposed an accurate general formula, applicable to most standard cross sections. with an axis of symmetry with respect to the axis of bending, given by:**

$$\frac{M_{Ed}}{M_{pl,Rd}} + \left(\frac{N_{Ed}}{N_{pl,Rd}}\right)^{\alpha,plan} = 1.0 \qquad \alpha_{plan} = 1.0 + 1.82\sqrt{\left(\frac{k}{w_{pl}} - 1.01\right)\frac{k-1}{w_{pl}-1}}.$$

- **$w_{pl} = W_{pl}/W_e$ is the ratio between the plastic bending modulus and the elastic modulus,**
- **k=v/i is the ratio between the maximum distance v from an extreme fiber to the elastic neutral axis and the radius of gyration i of the section about the axis of bending.**

Cross Section Resistance : M-N interaction

- **For a circular hollow section, the following exact expression may be established (Lescouarc'h, 1977): :**

$$M_{N,Rd} = M_{pl,Rd} \sin \frac{\pi(1-n)}{2}$$ where, $$n = N_{Ed} / N_{pl,Rd}$$

- **Interaction formulae for axial force and bi-axial bending have usually the following general format:**

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} = 1$$

For I or H cross sections subjected to N + My + Mz,

$$\alpha = \left(1.0 - 0.5\sqrt{n}\right)\alpha_{y,plan}$$

$$\beta = \frac{1+n}{1.0 - n^{(\alpha_{z,plan}-0.5)}},$$

For RHS cross subjected to N + My + Mz,

$$\alpha = \beta = \frac{1.7}{1-1.13\,n^2} \qquad (\text{if } n < 0.8);$$

$$\alpha = \beta = 6 \qquad (\text{if } n \geq 0.8).$$

Cross Section Resistance : Design Resistance

EC1993-1-1 Provisions

Clause 6.2.9 provides several interaction formulae between bending moment and axial force, in the plastic range and in the elastic range. These are applicable to most cross sections. But in all case the following shall be satisfied;

Class 1 or 2 sections

$$M_{Ed} \leq M_{N,Rd}$$

M_{Ed} is the design bending moment and $M_{N.Rd}$ represents the design plastic moment resistance reduced due to the axial force N_{Ed}

For rectangular solid sections under uni-axial bending and axial force, $M_{N,Rd}$is given by

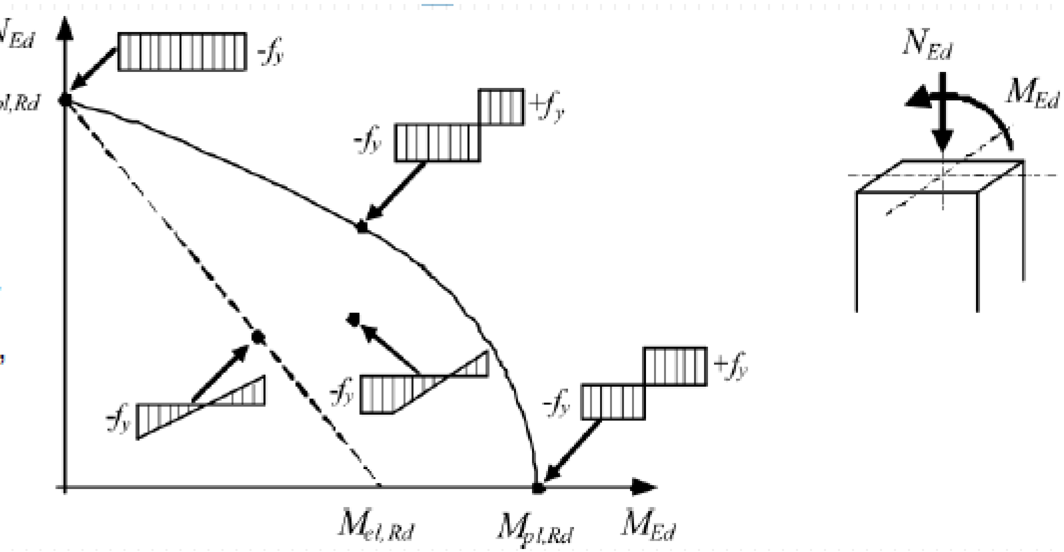
Cross Section Resistance : Design Resistance

For rectangular solid sections under uni-axial bending and axial force, $M_{N,Rdis}$ given by

$$M_{N,Rd} = M_{pl,Rd}\left[1-\left(\frac{N_{Ed}}{N_{pl,Rd}}\right)^2\right]$$

of

ant,



For low values of axial force, the reduction of the plastic moment resistance is not significant, as can be seen.

Cross Section Resistance : Design Resistance

For doubly symmetric I or H sections,

- It is not necessary to reduce the plastic moment resistance about y if the two following conditions are satisfied:

$$N_{Ed} \leq 0.25\, N_{pl,Rd} \quad \text{and} \quad N_{Ed} \leq 0.5\, h_w\, t_w\, f_y / \gamma_{M0}$$

- It is not necessary to reduce the plastic moment resistance about z if the following condition is verified:

$$N_{Ed} \leq h_w\, t_w\, f_y / \gamma_{M0}$$

For I or H sections, rolled or welded, with equal flanges and where fastener holes are not to be accounted for,

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5\,a} \quad \text{but} \quad M_{N,y,Rd} \leq M_{pl,y,Rd} \, ;$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \qquad \text{if } n \leq a \, ;$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \left[1 - \left(\frac{n-a}{1-a} \right)^2 \right] \qquad \text{if } n > a \, ,$$

where, $a = (A - 2bt_f)/A$, but $a \leq 0.5$.

For circular hollow sections,

$$M_{N,Rd} = M_{pl,Rd} \left(1 - n^{1.7} \right)$$

Cross Section Resistance : Design Resistance

For RHS of uniform thickness and for welded box sections with equal flanges and equal webs and where fastener holes are not to be accounted for,

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5\,a_w} \quad \text{but} \quad M_{N,y,Rd} \le M_{pl,y,Rd}$$

$$M_{N,z,Rd} = M_{pl,z,Rd} \frac{1-n}{1-0.5\,a_f} \quad \text{but} \quad M_{N,z,Rd} \le M_{pl,z,Rd}$$

where $a_w \le 0.5$ and $a_f \le 0.5$ are the ratios between the area of the webs and of the flanges, respectively, and the gross area of the cross section.

In a cross section under bi-axial bending and axial force, the N + M_y + M_z interaction can be checked by the following condition:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \le 1.0$$

where

α and β are parameters that depend on the shape of the cross section

I or H sections	$\alpha = 2;\ \beta = 5\,n$, but $\beta \ge 1$;
circular hollow sections	$\alpha = \beta = 2$;
rectangular hollow sections	$\alpha = \beta = \frac{1.66}{1-1.13\,n^2}$, but $\alpha = \beta \le 6$.

Cross Section Resistance : Design Resistance

Class 3 or 4 cross sections

In class 3 or 4 cross sections, the interaction between bending and axial force requires that the following condition be checked:.

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}}$$

where

$\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to bending moment and axial force, taking into account the fastener holes where relevant. It is calculated based on the gross cross section for class 3 cross sections, and on a reduced effective cross section for class 4 sections.

Interaction of bending, axial and shear force

The interaction between bending, axial and shear force should be checked as follows :

- When $V_{Ed} \leq$ % 50 of the design plastic shear resistance $V_{Pl,Rd}$, no reduction need be made in the bendin and axial force resistances

- When $V_{Ed} >$ % 50 of the design plastic shear resistance $V_{Pl,Rd}$, then the design resistance to the combination of bending moment and axial force should be calculated using a reduced yield strength for the shea area. This reduced strength is given by $(1-\rho)f_y$, where $\rho=(2\,V_{Ed}\,/\,V_{Pl,Rd}-1)^2$

Cross Section Resistance : Design Resistance

جَـامعة
المَـنارة
MANARA UNIVERSITY

Class 3 or 4 cross sections

In class 3 or 4 cross sections, the interaction between bending and axial force requires that the following condition be checked:.

$$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}}$$

where

$\sigma_{x,Ed}$ is the design value of the local longitudinal stress due to bending moment and axial force, taking into account the fastener holes where relevant. It is calculated based on the gross cross section for class 3 cross sections, and on a reduced effective cross section for class 4 sections.

Interaction of bending, axial and shear force

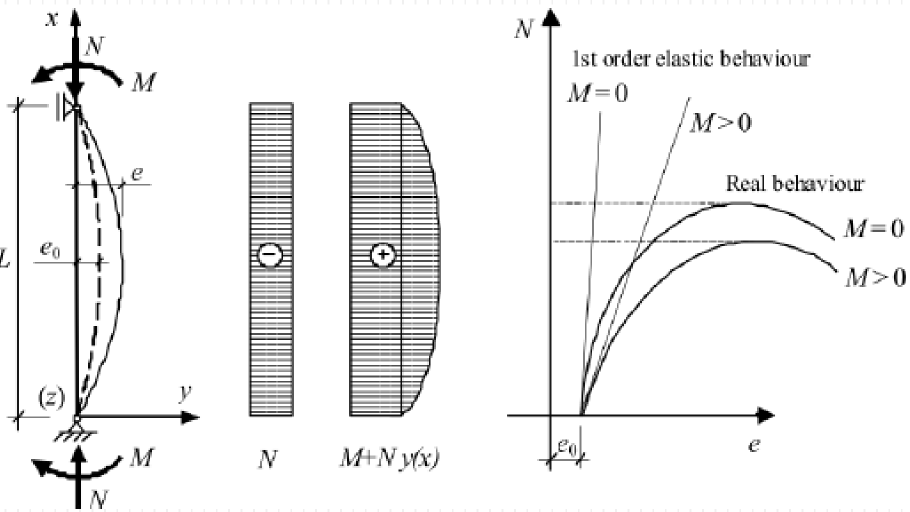
The interaction between bending, axial and shear force should be checked as follows :

- When $V_{Ed} \leq$ % 50 of the design plastic shear resistance $V_{Pl,Rd}$, no reduction need be made in the bendin and axial force resistances

- When $V_{Ed} >$ % 50 of the design plastic shear resistance $V_{Pl,Rd}$, then the design resistance to the combination of bending moment and axial force should be calculated using a reduced yield strength for the shea area. This reduced strength is given by $(1-\rho)f_y$, where $\rho=(2\,V_{Ed} / V_{Pl,Rd} -1)^2$

Buckling Resistance: Introduction

For a member under bending and compression, besides the first-order moments and displacements (obtained based or the undeformed configuration), additional second-order moments and displacements exist ("P-δ" effects); these should be taken into account.



- In the past, various interaction formulae have been propo to represent this situation over the full slenderness range.

- The present approach of EC3-1-1 is based on a linear-addi interaction formula, illustrated by expression:

$$f\left(\frac{N}{N_u},\frac{M_y}{M_{uy}},\frac{M_z}{M_{uz}}\right) \le 1.0$$

Where,

N, M_y and M_z are the applied forces and

N_u , M_{uy} and M_{uz} are the design resistances, that take in due acco the associated instability phenomena.

Buckling Resistance: Design Resistance

The development of the design rules, and in particular those adopted by EC3-1-1, is quite complex, as they have to incorporate;

- two instability modes, flexural buckling and lateral-torsional buckling (or a combination of both),
- different cross sectional shapes and several shapes of bending moment diagram, among other aspects.
- several common concepts, such as that of equivalent moment, the definition of buckling length and the concept of amplification.

Several procedures provided in EC3-1-1 were described for the verification of the global stability of a steel structure, including the different ways of considering the second order effects (local P-δ effects and global P- Δ effects).

This topic is solely focused on dealing with the second order effect arising from local P-δ effects.

Buckling Resistance: Design Resistance

Local P-δ effects are generally taken into account according to the procedures given in clause 6.3 of EC3-1-1

Clause 6.3.3(1) considers two distinct situations

Members not susceptible to torsional deformation, such as members of circular hollow section or other sections restrained from torsion. Here, flexural buckling is the relevant instability mode.	Members that are susceptible to torsional deformations, such as members of open section (I or H sections) that are not restrained from torsion. Here, lateral torsional buckling tends to be the relevant instability mode.

Buckling Resistance: Design Resistance

Members which are subjected to combined bending and axial compression should satisfy the following condition given in clause 6.3.3 of EC3-1-1

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$

About major axis y-y,

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$

About minor axis z-z,

Where,

N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ are the design values of the compression force and the maximum moments about the y-y and z-z axis along the member, respectively

$\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ are the moments due to the shift of the centroidal axis on a reduced effective class 4 cross section

χ_y and χ_z are the reduction factors due to flexural buckling

χ_{LT} is the reduction factor due to lateral torsional buckling

k_{yy}, k_{yz}, k_{zy}, k_{zz} are the interaction factors

Buckling Resistance: Design Resistance

Members which are subjected to combined bending and axial compression should satisfy the following condition given in clause 6.3.3 of EC3-1-1

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

About major axis y-y,

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

About minor axis z-z,

Where,

Values for $N_{Rk} = f_y A_i$, $M_{i,Rk} = f_y W_i$ and $\Delta M_{i,Ed}$

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y}\, N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z}\, N_{Ed}$

Buckling Resistance: Design Resistance-interaction factors

In EC3-1-1 two methods are given for the calculation of the interaction factors k_{yy} , k_{yz} , k_{zy} and k_{zz}.

Regardless of the method to be applied;

- In members that are not susceptible to torsional deformation, it is assumed that there is no risk of lateral torsional buckling (χ_{LT} = 1.0). And calculating the interaction factors k_{yy} , k_{yz} , k_{zy} and k_{zz} for a member not susceptible to torsional deformation.

Method 1, developed by a group of French and Belgian researchers,

According to this method , a member is not susceptible to torsional deformations if

- $I_T \geq I_Y$, or
- Incase $I_T < I_Y$, but the following condition is satisfied. $\bar{\lambda}_0 \leq 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$,

Where,

C_1 is a coefficient that depends on the shape of the bending moment diagram between laterally braced sections

$N_{cr,z}$ and $N_{cr,T}$ represent the elastic critical loads for flexural buckling about z and for torsional buckling, respectively

λ_0 is the non dimensional slenderness coefficient for lateral torsional buckling, assessed for a situation with constant bending moment.

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

Interaction factors	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	$C_{my} C_{mLT} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{yy}}$
k_{yz}	$C_{mz} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{yz}} 0.6\sqrt{\dfrac{w_z}{w_y}}$
k_{zy}	$C_{my} C_{mLT} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{zy}} 0.6\sqrt{\dfrac{w_y}{w_z}}$
k_{zz}	$C_{mz} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{zz}}$

Auxiliary terms:

$$\mu_y = \frac{1-\dfrac{N_{Ed}}{N_{cr,y}}}{1-\chi_y \dfrac{N_{Ed}}{N_{cr,y}}} \; ; \quad \mu_z = \frac{1-\dfrac{N_{Ed}}{N_{cr,z}}}{1-\chi_z \dfrac{N_{Ed}}{N_{cr,z}}} \; ; \quad w_y = \frac{W_{pl,y}}{W_{el,y}} \le 1.5 \; ; \quad w_z = \frac{W_{pl,z}}{W_{el,z}} \le 1.5$$

$$n_{pl} = \frac{N_{Ed}}{N_{Rk}/\gamma_{M1}} \; ; \quad a_{LT} = 1 - \frac{I_T}{I_y} \ge 0 \; ;$$

C_{my} and C_{mz} are factors of equivalent uniform moment, determined by the table on the slide # 26,

For class 3 or 4, consider $w_y = w_z = 1.0$.

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

$$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}},$$

where $b_{LT} = 0.5\, a_{LT}\, \bar{\lambda}_0^2 \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{M_{pl,z,Rd}}$.

$$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0.6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}},$$

where $c_{LT} = 10\, a_{LT} \frac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$.

$$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0.6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}},$$

where $d_{LT} = 2\, a_{LT} \frac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{C_{mz} M_{pl,z,Rd}}$.

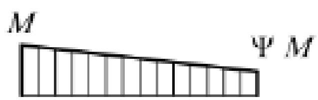
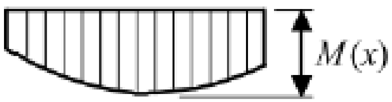
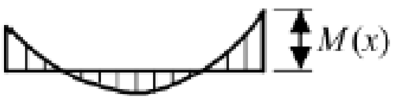
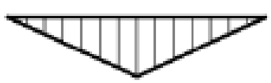
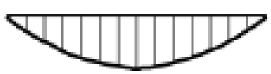
$$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) - e_{LT}\right] n_{pl} \geq \frac{W_{el,z}}{W_{pl,z}},$$ 4

where $e_{LT} = 1.7\, a_{LT} \frac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$.

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

Diagram of moments	$C_{mi,0}$
	$C_{mi,0} = 0.79 + 0.21\Psi_i + 0.36(\Psi_i - 0.33)\dfrac{N_{Ed}}{N_{cr,i}}$
 	$C_{mi,0} = 1 + \left(\dfrac{\pi^2 E I_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1\right)\dfrac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ according to the first order analyses $\lvert\delta_x\rvert$ is the maximum lateral deflection δ_z (due to $M_{y,Ed}$) or δ_y (due to $M_{z,Ed}$) along the member
 	$C_{mi,0} = 1 - 0.18\dfrac{N_{Ed}}{N_{cr,i}}$ $C_{mi,0} = 1 + 0.03\dfrac{N_{Ed}}{N_{cr,i}}$

Buckling Resistance: Design Resistance-interaction factors

Method 1, developed by a group of French and Belgian researchers,

Annex A of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 1

Auxiliary terms (continuation):

$\bar{\lambda}_{\max} = \max(\bar{\lambda}_y, \bar{\lambda}_z)$;

$\bar{\lambda}_0 =$ non dimensional slenderness for lateral torsional buckling due to uniform bending moment, that is, taking $\Psi_y = 1.0$ in Table 3.15;

$\bar{\lambda}_{LT} =$ non dimensional slenderness for lateral torsional buckling;

If $\bar{\lambda}_0 \le 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$: $C_{my} = C_{my,0}$; $C_{mz} = C_{mz,0}$; $C_{mLT} = 1.0$;

If $\bar{\lambda}_0 > 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$: $C_{my} = C_{my,0} + (1-C_{my,0})\frac{\sqrt{\varepsilon_y}\, a_{LT}}{1+\sqrt{\varepsilon_y}\, a_{LT}}$;

$$C_{mz} = C_{mz,0};\ C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}} \ge 1;$$

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A}{W_{el,y}}$ for class 1, 2 or 3 cross sections;

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A_{eff}}{W_{eff,y}}$ for class 4 cross sections;

$N_{cr,y}$ is the elastic critical load for flexural buckling about y;

$N_{cr,z}$ is the elastic critical load for flexural buckling about z;

$N_{cr,T}$ is the critical load for torsional buckling;

I_T is the constant of uniform torsion or St. Venant's torsion;

I_y is the second moment of area about y;

$C_1 = \left(\frac{1}{k_c}\right)^2$ where k_c is taken from Table 3.10.

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

According to Method 2, the following members may be considered as not susceptible to torsional deformation:

- members with circular hollow sections (CHS).
- members with rectangular hollow sections (RHS) (there is widly argued exception to this rule presented in (
- members with open cross section, provided that they are torsionally and laterally restrained.

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Interaction factors k_{ij} in members not susceptible to torsional deformations according to Method 2

Interaction factors	Typeof section	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	I or H sections and rectangular hollow sections	$C_{my}\left(1+0.6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0.6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{my}\left(1+0.8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I or H sections and rectangular hollow sections	k_{zz}	$0.6\,k_{zz}$

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Interaction factors k_{ij} in members not susceptible to torsional deformations according to Method 2

k_{zz}	I or H sections	$C_{mz}\left(1+0.6\bar{\lambda}_z\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(2\bar{\lambda}_z-0.6\right)\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+1.4\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
	rectangular hollow sections	$\leq C_{mz}\left(1+0.6\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(\bar{\lambda}_z-0.2\right)\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$ $\leq C_{mz}\left(1+0.8\dfrac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$

In I or H sections and rectangular hollow sections under axial compression and uniaxial bending ($M_{y,Ed}$), k_{zy} may be taken as zero.

Buckling Resistance: Design Resistance-interaction factors

جَـامعة المَـنارة

MANARA UNIVERSITY

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Interaction factors k_{ij} in members susceptible to torsional deformations according to Method 2

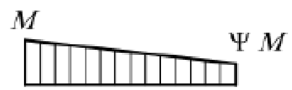
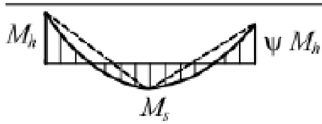
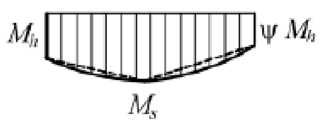
Interaction factors	Elastic sectional properties (Class 3 or 4 sections)	Plastic sectional properties (Class 1 or 2 sections)
k_{yy}	k_{yy} of Table 3.16	k_{yy} of Table 3.16
k_{yz}	k_{yz} of Table 3.16	k_{yz} of Table 3.16
k_{zy}	$\left[1-\frac{0.05\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ $\geq\left[1-\frac{0.05}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$	$\left[1-\frac{0.1\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ $\geq\left[1-\frac{0.1}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ for $\bar{\lambda}_z<0.4:\ k_{zy}=0.6+\bar{\lambda}_z$ $\leq 1-\frac{0.1\bar{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}$
k_{zz}	k_{zz} of Table 3.16	k_{zz} of Table 3.16

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

Equivalent factors of uniform moment C_{mi} according to Method 2

Diagram of moments	Range		C_{my}, C_{mz} and C_{mLT}	
			Uniform loading	Concentrated load
	$-1 \le \Psi \le 1$		$0.6 + 0.4\Psi \ge 0.4$	
 $\alpha_s = M_s / M_h$	$0 \le \alpha_s \le 1$	$-1 \le \Psi \le 1$	$0.2 + 0.8\alpha_s \ge 0.4$	$0.2 + 0.8\alpha_s \ge 0.4$
	$-1 \le \alpha_s < 0$	$0 \le \Psi \le 1$	$0.1 - 0.8\alpha_s \ge 0.4$	$-0.8\alpha_s \ge 0.4$
		$-1 \le \Psi < 0$	$0.1(1 - \Psi) - 0.8\alpha_s \ge 0.4$	$0.2(-\Psi) - 0.8\alpha_s \ge 0.4$
 $\alpha_h = M_h / M_s$	$0 \le \alpha_h \le 1$	$-1 \le \Psi \le 1$	$0.95 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \Psi \le 1$	$0.95 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
		$-1 \le \Psi < 0$	$0.95 + 0.05\alpha_h(1 + 2\Psi)$	$0.90 + 0.10\alpha_h(1 + 2\Psi)$

In the calculation of α_s or α_h parameters, a hogging moment should be taken as negative and a sagging moment should be taken as positive.

Buckling Resistance: Design Resistance-interaction factors

Method 2, developed by a group of Austrian and German researchers,

Annex B of EC3-1-1 presents Tables, for the calculation of the interaction factors according to Method 2

For members with sway buckling mode, the equivalent uniform moment factor should be taken as $C_{my} = 0.9$ or $C_{mz} = 0.9$, respectively.

Factors C_{my}, C_{mz} and C_{mLT} should be obtained from the diagram of bending moments between the relevant braced sections, according to the following:

Moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

Equivalent factors of uniform moment C_{mi} according to Method 2

Design According to EC3:
Section classification for sections under bending and axial force

According to EC3, the classification of a cross section is based on its maximum resistance to the type of applied internal forces, independent from their values.

- This procedure is straightforward to apply for cross sections subjected to either bending or compression.
- However, the presence of both the compression and bending moment on the cross-section member, generates a stress distribution between that related to pure compression and that associated with the presence of the sole bending moment.
- Bearing in mind this additional complexity, simplified procedures are often adopted, such as:

i. to consider the cross section subjected to compression only, being the most unfavourable situation (too conservative in some cases)

ii. to classify the cross section based on an estimate of the position of the neutral axis based on the applied internal forces.

- In the later case the neutral axis depth depends on whether the section can plastify, the bending axis, the section profile.

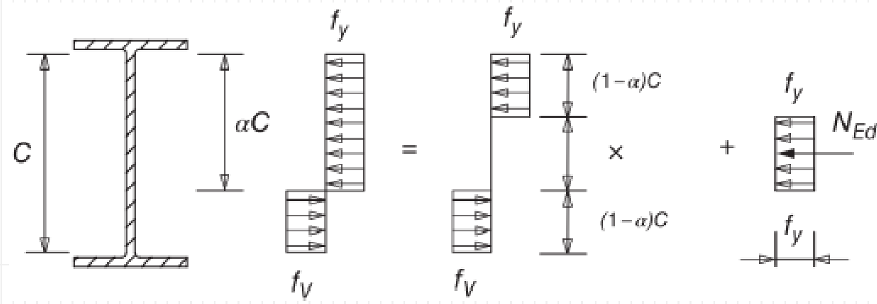
Design According to EC3:
Section classification for sections under bending and axial force

For Bending and Compression about a strong Axis (y-y).

Normal stress distribution on the web depends on the value of the design axial load by means of parameter **α** for profiles able to resist in the plastic range (classes 1 and 2).

Applying Section Equilibrium and Super positioning

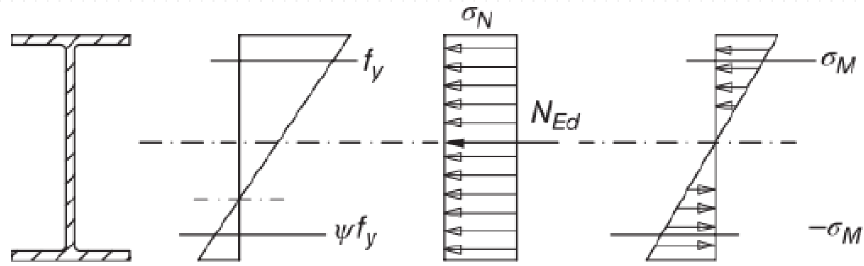
$$\alpha = \frac{1}{2}\left(1 + \frac{1}{c} \cdot \frac{N_{Ed}}{t_w f_y}\right)$$



in case of elastic normal stress distribution, reference has to be made to parameter **ψ** (classes 3 and 4).

Applying Section Equilibrium and Super positioning

$$\psi = 2\frac{N_{Ed}}{A\,f_y} - 1$$



With reference to the case of a neutral axis located in the web, α ranges between 0.5 (bending) and 1 (compression) and ψ ranges between −1 (bending) and 1 (compression).

Once the stress distribution is assumed and the values of α and ψ can be used to classify the section using tables 5.2 (sheet1 through 3)